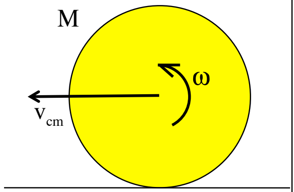
Problem 10.55



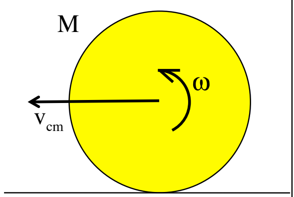
This problem is designed to point out one thing and one thing only. An object that is rotating as it moves across a surface has both *rotational kinetic energy* and *translational kinetic energy*. Starting with the translational KE:

$$\begin{aligned} KE_{trans} &= \frac{1}{2}m\left(v_{cm}\right)^2 \\ &= \frac{1}{2}(10.0\ \text{kg})(10.0\ \text{m/s})^2 \\ &= 500.\ \text{J} \end{aligned}$$

As for rotational KE, we know the angular speed of the object as it rotates about its center of mass is:

$$\omega = \frac{v_{cm}}{R}$$

(For a justification of this, see note at end of problem.)



We also know the *moment of inertia* of a disk is:

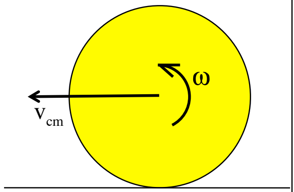
$$I_{disk} = \frac{1}{2}MR^2$$

So putting it all together, the *rotational kinetic energy* becomes:

$$\begin{aligned} KE_{trans} &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(\frac{1}{2}m\cancel{R^2}\right)\left(\frac{v_{cm}}{\cancel{R}}\right)^2 \\ &= \frac{1}{4}m\left(v_{cm}\right)^2 \\ &= \frac{1}{4}(10.0\ \text{kg})(10.0\ \text{m/s})^2 \\ &= 250.\ \text{J} \end{aligned}$$

The total kinetic energy is just the sum of the two, or 750. J.

DON'T GO BEYOND THIS POINT UNLESS YOU ARE FEELING NERDY AND HAVE TIME.



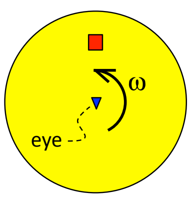
You are done with the problem. This last part is extra and is for those who really want to understand why what you've done makes sense.

So why is it that the velocity of a rolling object's *center of mass* and the *angular speed* of the mass in the object *about* that center of mass are related by:

$$v_{cm} = R\omega?$$

After all, both variables seem to be attached to the same point—the *center of mass*. The answer is the consequence of a confluence of two odd but true observations, all of which I will expose by the numbers.

1.) Part of the appeal of using rotational parameters when dealing with rolling situations is in found in the fact that if you know, say, the *angular speed* of the mass in the structure as it rotates about the structure's *center of mass*, you know the *angular speed* of the mass in the structure as it is viewed to rotate about *any point on the structure*.



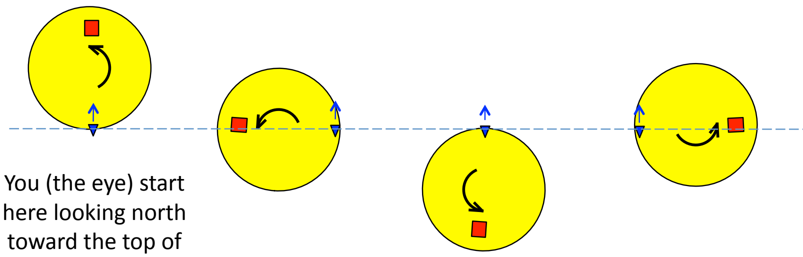
This rather obscure observation can be seen if you view *one revolution* of a pinned disk from two different perspectives. Specifically:

You are sitting just above the center of the disk as its mass rotates in a circle underneath you (see the eye). You pick a point on the disk (the red box), and when it is opposite you you start your stopwatch. Some time "t" passes before the point gets back in front of you. You deduce that the angular speed of the rotation from your perspective is two-pi radians divided by "t," or

$$\omega = \frac{2\pi}{t}$$

You now mix things up a bit with an unusual set-up. You again sit above the disk but this time, do it at one edge, *always facing north*. Although you move *with* the disk as it rotates, you think of yourself as the center of the universe and, hence, don't register that motion. Instead, you think you are sitting still with everything moving around underneath you. What is the disk's angular speed as it appears to be rotating around underneath you?

A sketch of the situation will make this more obvious.



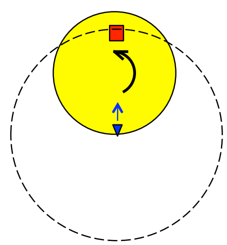
You (the eye) start here looking north toward the top of the page at the red box

From the eye's perspective, as the rotation proceeds "underneath it," it hasn't moved and it is still looking north, but where does the box appear to have gone?

More time passes. Now where is the box?

And now?

From the perspective of the eye, the box followed a circular path around it, as shown to the right.



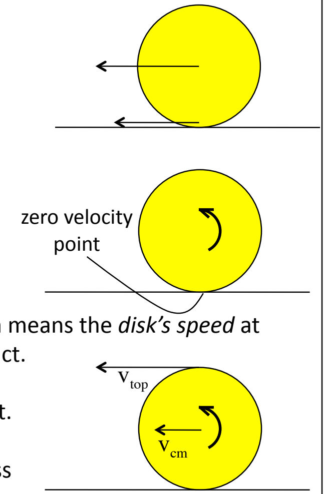
So lets say you were the eye and your friend was the gal sitting over the disk at its center of mass. It takes "t" seconds for both of you to observe the box to move in a circle around you, and the *angular velocity* of the mass moving underneath you would, in both cases, be:

$$\omega = \frac{2\pi}{t}$$

Bottom line: If you know the *angular speed* of a rotating mass relative to one point on the mass, you know it relative to ALL points on the mass as they will all be the same.

2.) The second bit of information you need is equally bizarre.

Let's say you have a disk that is sliding over a table (it could be both sliding and rolling, but it hasn't completely caught traction). Clearly the speed of the disk's *contact point relative to the table* would NOT be zero. The table top would be stationary (no speed) while the disk would be sliding and would *have* speed. (See first sketch.)



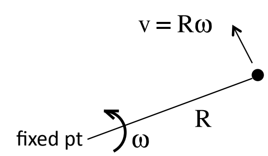
So what is the situation when the disk is rolling *without* slipping? In that case, the *contact point's speed* and the *table's speed* must be the same, which means the *disk's speed* at the contact point *must be zero* at the instant of contact.

This isn't *that* hard to believe if you just think about it. Instantaneously, is the mass at the *center of mass* moving with the same translational speed as the mass at the top of the disk? NO! In fact, it's moving half as fast (again, see sketch).

Bottom line for this bit of insanity? The *contact point velocity* for a rolling object is ZERO.

So now we are ready to justify

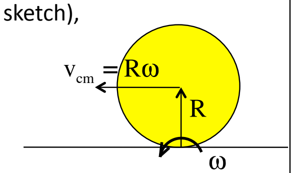
$$v_{cm} = R\omega.$$



We know that when an object rotates about a fixed point, we can relate the object's *angular speed* about that point to its *translational speed* some distance "R" units away. The string situation to the right is the classic example.

So think about our rolling disk? It's contact point is a fixed point; its angular speed is the same everywhere, including about the contact point. The velocity of the *center of mass*, following the string logically (see sketch), must be:

$$v_{cm} = R\omega.$$



FINITO! (And if you got through all of that without going blind, you are a whole lot cooler than I was at your age.)